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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 9 | a) | The grades of a class of 9 number of students on a midterm report (x) and on the final examination (y) are as follows:   |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | x | 77 | 50 | 71 | 72 | 81 | 94 | 96 | 99 | 67 | | y | 82 | 66 | 78 | 34 | 47 | 85 | 99 | 99 | 68 |   Calculate the correlation coefficient. | 2 |
|  | b) | Estimate the regression line from 9(a). | 2 |
|  | c) | Estimate the final examination grade of a student who received a grade of 85 on the midterm report in 9(a). | 2 |
| 10 | a) | The grades in a statistics course for a particular semester were as follows:  **Grade A B C D F**  ***f* 14 18 32 20 16**  Using a suitable statistical method, test the hypothesis, at the 0.05 level of significance, that the distribution of grades is uniform. | 2 |
|  | b) | In an experiment, to study the dependence of hypertension on smoking habits, the following data were taken on 180 individuals.   |  |  |  |  | | --- | --- | --- | --- | |  | Non smokers | Moderate smokers | Heavy smokers | | Hypertension | 21 | 36 | 30 | | No hypertension | 48 | 26 | 19 |   Write the critical region at 0.05 level of significance. | 2 |
|  | c) | With reference to 10(b), test the hypothesis, that the presence or absences of hypertension is independent of smoking habits. | 2 |
|  |  | \*\*\*\*Best of Luck\*\*\*\* |  |

**END SEMESTER EXAMINATION, APRIL-2018**

**PROBABILITY & STATISTICS (MTH-2002)**

**Programme: B.Tech Semester : 4th**

**Full Marks: 60 Time: 3 Hours**

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| **Subject/Course Learning Outcome** | **\*Taxonomy**  **Level** | **Ques.**  **Nos.** | **Marks** |
| Apply probability axioms to compute probability and conditional probability | L3,L3,L4,L3 | 1(a,b), 2(a) | 2\*3 |
| Define random variables and compute probability distributions, joint & marginal distribution | L4,L4,L3,L5,L5 | 1(c), 2(b,c), 3(a,b,c) | 2\*6 |
| Compute expectation of random variables and their functions and compute moments and moment generating functions of a random variable | L3,L4,L4  L3,L4 | 4(a,b,c),7(a),6(c) | 2\*5 |
| Discuss discrete probability distribution viz: Binomial, Poisson & Hypergeometric and continuous probability distribution distributions viz: Uniform, Normal Gamma & Exponential | L3,L4, L4, L4 | 5(a,b,c), 6(a,b), | 2\*5 |
| Estimate the population mean and variance of a normal distribution by point and interval estimation | L4 | 7(b,c) | 2\*2 |
| Infer about population parameter through hypothesis testing with the help of a random sample | L1,L4,L3,L4,L4, L3,L4,L4 | 8(a,b,c), 10(a,b,c), | 2\*6 |
| Analyze linear regression and co-relation | L3,L5,L5 | 9(a,b,c) | 2\*3 |

\*Bloom’s taxonomy levels: Knowledge (L1), Comprehension (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

**Answer all questions. Each question carries equal mark.**

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | a) | An experiment consists of tossing a die and then flipping a coin once if the number on the die is even. If the number on the die is odd, the coin is flipped twice. Describe the sample space using trees. | 2 |
|  | b) | If 3 books are picked at random from a shelf containing 7 mathematics, 3 physics and 2 chemistry books, calculate the probability that 2 mathematics books are selected. | 2 |
|  | c) | Verify the following distribution is a valid density function. | 2 |
| 2 | a) | In a certain assembly plant, three machines, ***B*1, *B*2, and *B*3**, make **30%, 45%, and 25%,** respectively, of the products. It is known from past experience that **2%, 3%, and 2%** of the products made by each machine, respectively, are defective. Apply conditional probability to find that a randomly selected product is defective? | 2 |
|  | b) | With reference to 2(a), if a product was randomly chosen and found to be defective, compute the probability that it was made by machine B3. | 2 |
|  | c) | For the following probability distribution of the discrete random variable *X,* compute the value of ‘***c***’: . | 2 |
| 3 | a) | If the joint probability distribution of X & Y is given by find the value of **‘c’.** | 2 |
|  | b) | With reference to 3(a), evaluate the marginal distributions of the random variables X & Y. | 2 |
|  | c) | From 3(a), calculate P(X >2, Y ≤ 1) | 2 |
| 4 | a) | A coin is biased such that a head is three times as likely to occur as a tail. Calculate the expected number of tails when this coin is tossed twice. | 2 |
|  | b) | Suppose X is a random variable with probability density function given by  Compute the variance of Z = 2X + 1. | 2 |
|  | c) | A random variable X has a mean µ = 10 and variance σ2 = 4. Using Chebyshev’s theorem, find P (5 < x < 15). | 2 |
| 5 | a) | The average number of field mice per acre in a 5-acre wheat field is estimated to be 12. Compute the probability that fewer than 7 field mice are found on a given acre. | 2 |
|  | b) | In an NBA championship series, the team that wins three games out of five is the winner. Suppose that teams *A* and *B* face each other in the championship games and that team *A* has probability 0.55 of winning a game over team *B*. Calculate the probability that team *A* would win the series? | 2 |
|  | c) | Suppose X follows a continuous uniform distribution from 1 to 5. Determine the conditional probability . | 2 |
| 6 | a) | A process yields 10% defective items. If 100 items are randomly selected, calculate the probability that the number of defectives is less than 8 by using the normal approximation to the Binomial distribution. | 2 |
|  | b) | If the random variable X, has a Gamma distribution with and . Evaluate. | 2 |
|  | c) | Let X be a random variable with probability distribution    Calculate the probability distribution of Y = X2. | 2 |
| 7 | a) | Derive the moment generating function for Poisson distribution. | 2 |
|  | b) | Compute the maximum likelihood estimator for ‘µ’ of normal population from the sample of observations **x1, x2, … ,xn**. | 2 |
|  | c) | The length of life of light bulbs that is approximately normally distributed with a standard deviation of 40 hours. If a sample of 30 bulbs has an average life of 780 hours, estimate a 96% confidence interval for the population mean of all bulbs. | 2 |
| 8 | a) | A random sample of 64 bags of white cheddar popcorn weighed on average 5.23 ounces with a standard deviation of 0.24 ounce. Test the hypothesis that µ=5.5 against the alternative hypothesis µ < 5.5 at 0.05 level of significance. | 2 |
|  | b) | A soft-drink machine is said to be out of control if the variance of the contents exceeds 1.15 deciliters. A random sample of 25 drinks from this machine has variance 2.03 deciliters. Use α =0.05 and explain the critical region (Assume normal distribution). | 2 |
|  | c) | With reference to 8(b), test whether the machine is out of control. | 2 |